MOHAMMED GUEDIRI

Erratum to “On the geodesic connectedness of simply connected Lorentz surfaces”


<http://www.numdam.org/item?id=AFST_2001_6_10_1_33_0>
Erratum to “On the geodesic connectedness of simply connected Lorentz surfaces”

MOHAMMED GUEDIRI (1)

In the statements of Proposition 2.2, Theorem 3.2 (the main result) and Theorem 3.3 in our paper [3], the word “globally hyperbolic” should be replaced by “strongly globally hyperbolic”, where a Lorentz surface \((M, g)\) is said to be strongly globally hyperbolic if both \((M, g)\) and \((M, -g)\) are globally hyperbolic and any two distinct points of \(M\) are causally related in either \((M, g)\) or \((M, -g)\).

Without this strong assumption, one can provide counterexamples to Theorem 3.2. Consider for instance the universal covering of the two-dimensional anti-de Sitter space. This covering may be represented by the strip

\[
\widetilde{H}_2^1 = \{(x, y) / -\pi/2 < x < \pi/2\}
\]

in \(\mathbb{R}^2\) endowed with the Lorentz metric \(g = \sec^2 x \left(dx^2 - dy^2\right)\).

It is well known that this space is not geodesically connected (cf. [1], pp. 199-200), and by drawing the two transversal families of null geodesics (called null foliations) one can easily check that \(\left(\widetilde{H}_2^1, g\right)\) is not globally hyperbolic. However, the same diagram shows that the Lorentz surface \(\left(\widetilde{H}_2^1, -g\right)\) is globally hyperbolic. This gives a simply connected Lorentz surface which is globally hyperbolic but geodesically disconnected.

In the compact case, inspection of the example given in [2], pp. 120-121 shows that the universal covering of the torus \(T^2\) endowed with the Lorentzian metric \(g = 2 \sin x dx dy + \cos^2 x \left(dx^2 - dy^2\right)\) is globally hyperbolic.

---

(1) Department of Mathematics, College of Sciences, King Saud University, P.O. Box 2455, Riyadh 11451, Saudi Arabia. E-mail: mguediri@ksu.edu.sa

2000 Mathematics Subject Classification. Primary 53C50; Secondary 53C22.

Key words and phrases. Geodesic connectedness, global hyperbolicity.
but fails to be geodesically connected. Hence, global hyperbolicity does not imply geodesic connectedness of simply connected Lorentz surfaces even in the compact case.

Bibliography

