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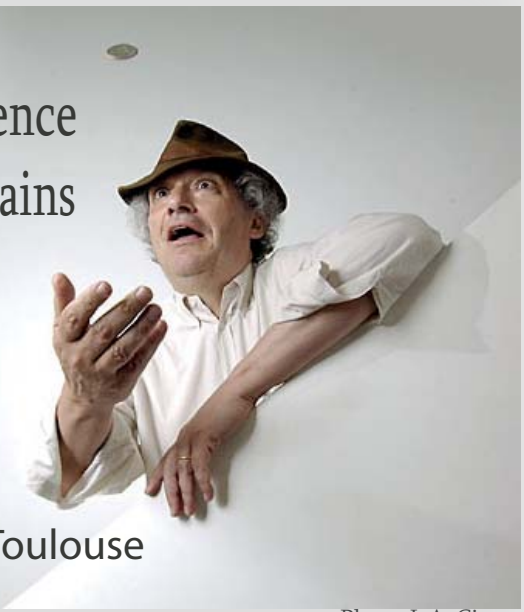
Talking Across Fields

A workshop on the convergence to equilibrium of Markov chains around Persi Diaconis



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March 24th-28th 2014

Institut de Mathématiques de Toulouse
Amphithéâtre Schwartz

Pietro Caputo *Università Roma Tre*
Patrick Cattiaux *Université Toulouse III*
Djalil Chafaï *Université Paris-Dauphine*
Thierry Delmotte *Université Toulouse III*
Persi Diaconis *Stanford University*
Randal Douc *Telecom SudParis*
Martin Dyer *University of Leeds*
Shayan Oveis Gharan *Stanford University*
Jim Hobert *University of Florida*
Mark Jerrum *Queen Mary University of London*
Wilfrid Kendall *University of Warwick*
Werner Krauth *Ecole Normale Supérieure*
Eyal Lubetzky *Microsoft Research*
Florent Malrieu *Université de Tours*
Fabio Martinelli *Università Roma Tre*
Eric Moulines *Telecom Paris-Tech*
Yann Ollivier *Université Paris-Sud*
Yuval Peres *Microsoft Research*
Clément Rau *Université Toulouse III*
Laurent Saloff-Coste *Cornell University*
Prasad Tetali *Georgia Institute of Technology*

Scientific organizer Laurent Miclo
www.math.univ-toulouse.fr/Talking_Across_Fields/index.php

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Preface

This special volume of the Annales de la Faculté des Sciences de Toulouse edits the proceedings of the conference **Talking Across Fields** which took place in Toulouse from Monday 24th to Friday 28th of March 2014. It was one of the events organized during the semester invitation of Persi Diaconis to the Institut de Mathématiques de Toulouse by the Labex Centre International de Mathématiques et d'Informatique.

The subject of this conference was the convergence to equilibrium of Markov chains and related fields, the objective being to review its latest advances as well as various directions followed by its applications. Indeed one of the characteristic of the meeting was to gather experts with different scientific backgrounds, as it is emphasized by the name of the conference and by the presentation written by Persi Diaconis:

A variety of fields study rates of convergence of Markov Chains to their stationary distribution. This includes probabilists, statisticians, analysts, computer scientists and physicists. The conference will aim to bring together people working in these disciplines and try to encourage them to talk to one another. Each field has its own techniques and by now these have developed quite separately. For example, they tend not to cite one another! Of course, the different fields have slightly different problems but this is all grist for the mill.

The multidisciplinarity aspect of the conference is equally reflected by the list of its speakers which is to be find in the poster before this preface. Several of them agreed to submit a paper for the present special edition of the Annales de la Faculté des Sciences de Toulouse. Let us succinctly present these contributions and how they are related.

The volume starts with a historical perspective on the concept of entropy by **Djalil Chafaï**. Primary emphasis is put on the entropy production by evolution equations in various fields, since this is strongly related to the subject of the conference. This leisurely and very inspiring survey begins with the H-theorem of Boltzmann and the free entropy of Helmholtz. The entropy reappears in the approach of Shannon to the central limit theorem and in its free probability analogue due to Voiculescu. The limiting circular law of the complex Ginibre ensemble enables a transition toward the recent use of entropy in study of particles in Coulomb or Riesz repulsive interactions.

The following paper of **Pietro Caputo**, **Georg Menz** and **Prasad Tetali** also deals with entropy, providing its approximate tensorization under some weak dependence assumptions, suitable for the study of the trend to equilibrium of certain interacting particle systems. The tensorization properties of entropy are important in statistical physics and in information theory. They allow to control the disorder of a compound system with the partial disorder of its components.

We stay in the realm of dynamical statistical physics with the two next papers. **Paul Chleboun**, **Alessandra Faggionato** and **Fabio Martinelli** study the d-dimensional East process, an interacting particle system belonging to the class of kinetically constrained spin models. Due to the constraints, establishing convergence to equilibrium is a difficult task, in particular the classical functional inequalities techniques do not apply. A new approach is developed to prove a local form of convergence to equilibrium and to deduce the scaling of the mixing time and the logarithmic Sobolev constant in a box in terms of its size.

In a series of important articles, **Eyal Lubetzky** and **Allan Sly** have recently introduced a new approach, called information percolation, to solve the cutoff problem for the Glauber dynamics of the Ising model all the way down to the critical temperature. In the present expository paper, the authors make the most welcome effort to explain how to use this theory in the setting of very high temperature Ising model on $\mathbb{Z}^d$. In this simplified context, their analysis of the underlying space-time branching process associated to the graphical representation of the Glauber dynamics is very illuminating.

The cut-off phenomenon is also at the heart of the paper of **Yuval Peres** and **Perla Sousa**, who give the first example of a lazy random walk on a tree for which the mixing time is not equivalent to the relaxation time. These quantities, as well as the window of the corresponding cut-off phenomenon, are estimated. The model is a good illustration of the probabilistic techniques used to deduce the cut-off phenomenon on trees.

We keep on in the company of traditional finite Markov chains with the work of **Matthias Erbar**, **Jan Maas** and **Prasad Tetali**, developing the discrete notion of Ricci curvature, applicable to stationary distributions of continuous time Markov chains, based on entropy flows. This approach was recently introduced by the second author. In particular, they compute the Ricci curvature lower bound for two very classical examples of random walks, namely, a chain on a slice of the n-dimensional discrete cube (the so-called Bernoulli-Laplace model) and the random transposition shuffle of the symmetric group of permutations on n letters.

Beside Ricci curvature, another tool coming from geometrical analysis to investigate finite Markov chains is the resort to spectral decompositions and their links with Cheeger inequalities. Thus **Shayan Oveis Gharan** provides an overview of several important and recent advances in spectral graph theory concerning estimates of higher order eigenvalues, especially describing their interactions with topics in both pure mathematics and computer sciences. The emphasis is put on the spectral embedding of graphs in Euclidean spaces via the first eigenvectors, which is the main tool behind the new bounds.

A different point of view on isoperimetry is proposed by **Thierry Delmotte** and **Clément Rau**, who update their unpublished preprint on the anchored or rooted isoperimetric inequality (considering sets containing a fixed point) and its application to random walks. In particular it contains a new and much simpler proof for a version of a theorem of Virag.

Random walks are also the subject of the article of **Laurent Saloff-Coste** and **Tianyi Zheng**, providing a sharp lower bound for the return probability on a finitely generated group of polynomial volume growth. The originality of this work is that the driving symmetric measure has infinite moments of all orders. It leads to new results improving the understanding of random walks.

A different kind of Markov evolution is presented by **Florent Malrieu**: Piecewise Deterministic Markov Processes and their sometimes surprising long time behavior. These processes have attracted a lot of attention recently and only quite simple examples are considered in this paper, whose goal is to illustrate the combined use of tools such as coupling methods, spectral decomposition, PDE techniques and functional inequalities. Some challenging open questions end this stimulating exposition.

The protean investigation of convergence to equilibrium often feeds on examples, this is further illustrated by the two following papers, dealing with particular and instructive examples. **Martin Dyer** and **Haiko Müller** focus on the switch Markov chain for sampling from perfect matchings of a bipartite graph, where a single step of the Markov chain takes two edges in a perfect matching and replaces them by pairing the edges' endpoints in a different way. Following a previous paper of Diaconis, Graham and Holmes, they investigate the computational complexity and the mixing time on certain classes of graphs. It leads to a challenging conjecture concerning Hamilton cycles in so-called monotone graphs.

James Hobert and **Kshitij Khare** study the rate of convergence to equilibrium for a Gibbs sampler Markov chain introduced recently by Jovanovski and Madras. The resulting Wasserstein bound enables to generalize a result due to Diaconis, Khare and Saloff-Coste.

Sampling computational Markov chains are heavily used in physics. Thus **Michel Caffarel** introduces several Monte Carlo algorithms and importance sampling techniques to improve them, used by the physicists in order to simulate ground states. When one is considering Fermionic ground states, the famous sign problem occurs. It is one of the fundamental challenge in present computational physics.

Finally with **Persi Diaconis**, we consider the quantitative convergence to quasi-stationarity for absorbing, finite, irreducible Markov processes. Using Doob transforms, we come back to the well-studied situation of convergence to equilibrium of ergodic Markov processes, subject of the conference. Several examples illustrate the obtained explicit estimates.

Before the reader chooses among the articles s/he is more interested in, it is now time for the acknowledgments. Obviously, they are fully deserved by all the above-mentioned people and institutions who made possible the publication of this volume. But the not-mentioned referees should not be forgotten, two of them were required by each article and the influence of their reports extended to the writing of this introduction. So let me finish by warmly thanking all of them again for their generous and constructive help.

Laurent Miclo

Toulouse, Friday 10th of June 2015