## ANNALES DE LA FACULTÉ DES SCIENCES TOULOUSE Mathématiques

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## Preface

This volume assembles a collection of expository articles based on some of the courses delivered in Toulouse from February to May 2014, as part of a program entitled *Trimestre thématique EDP et Probabilités*<sup>(1)</sup> and concentrated on the interactions between probability and partial differential equations. These courses do not cover the full scope of the program, but nevertheless present a quite large overview of the topics involved.

Interplay between probability theory and partial differential equations has a long standing history, perhaps going back to Einstein's 1905 paper on Brownian motion. On the one hand, the law of many natural random objects (Markov diffusion processes for example) are given through solutions of partial differential equations (mostly of parabolic nature), and on the other hand, many natural equations, arising for example from fluid mechanics or statistical physics, have natural probabilistic representations.

At the present time, these topics are becoming more and more involved. First, one is led to study specific partial differential equations, linear or not, in which randomness appears either in the initial data, or in the evolution process such as random forces. As such, stochastic partial differential equations (such as KPZ,  $\Phi_d^4$  fields, to name but a few) are under rapid development, and require sophisticated tools such as regularity structures.

Randomness may also appear at another level, looking at partial differential equations from a geometric measure theory point of view. Indeed, even first order differential equations (evolutions of particles, integration of vector fields) may be studied in this context, where random vector fields or random loops are in general not smooth and require for their study various tools of probability theory. BV vector fields, rough path approaches, point particle approximations, are some of the many tools which are used in this context and exposed in this volume.

Another important aspect is data assimilation, where one has to couple evolution equations (arising from fluid mechanics or geosciences) with the observed data, in particular for the purpose of weather forecast. This may

<sup>&</sup>lt;sup>(1)</sup> This thematic program took place within the activities of the LabEx CIMI (Centre International de Mathématiques et d'Informatique de Toulouse) and was partially supported by ANR-11-LABX-0040-CIMI within the program ANR-11-IDEX-0002-02. The organizers thank the CIMI staff for their invaluable help during this event.

be sought as part of nonlinear filtering theory, where the random unknown state of a fluid has to be conditioned with the help of the observation. This requires an important algorithmic effort, since those laws, when discretized, are high dimensional and may not be handled by usual methods.

Finally, convergence to equilibrium and long time behavior are often controlled through precise functional inequalities, which in turn may be studied through evolution equations.

The choice of the talks presented here aims at providing an overview on these different aspects.

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