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*Analysis of Monge–Ampère, a tribute to Ahmed Zeriahi*

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## Preface

The volume gathers some articles that were presented at the conference AMAZER held in Toulouse in June 2021, in honor of Ahmed Zeriahi on the occasion of his retirement. AMAZER is an acronym for *Analysis of Monge–Ampère, a tribute to Ahmed Zeriahi*. It is also reminiscent of “Amazigh”, the name of the Berber people, which means “free man” in tamazight.

Ahmed spent most of his career at the University Paul Sabatier in Toulouse, where he arrived in 1973 after completing his baccalaureate in Rabat (Morocco). He defended his “doctorat d’état” in 1986 under the supervision of Nguyen Van Thanh. He took a position as Professor at the University of Rabat at the end of the 80s, before returning to Toulouse as full Professor from 1992 until his retirement in September 2020.

Most of Ahmed’s work is devoted to complex analysis in several variables and its many applications to dynamical systems and geometry. He has become a leading expert in *pluripotential theory*, establishing fine properties of the complex Monge–Ampère operator which plays a central role in complex analytic geometry.

Ahmed has supervised a dozen of PhD theses of French, Moroccan and Vietnamese students. He has established friendships and rich collaborations with researchers from many European countries (France, Italy, Iceland, Poland, Sweden), and has worked hard to develop exchanges with countries in the process of scientific development (Morocco, Uzbekistan, Senegal, Turkey, Vietnam).

His good mood, his generosity and his energy have greatly contributed to the success of these scientific exchanges. Many authors (in particular some of his many collaborators!) wished to pay tribute to him, leading to this volume. Let us now briefly describe the contributions it contains.

*Berman and Berndtsson* establish Moser–Trudinger type inequalities for Hodge classes on a compact Kähler manifold, positively answering a conjecture of Aubin. They also study the local setting of bounded pseudoconvex domains in  $\mathbb{C}^n$  in connection with a Brezis–Merle inequality of Ahmed and collaborators, obtaining a sharp Moser–Trudinger inequality for  $S^1$ -invariant plurisubharmonic functions

*Boucksom and Jonsson* establish the foundations of a global non-archimedean pluripotential theory, generalizing the work of Ahmed and his collaborators in the framework of Berkovich geometry over a trivially valued

field. They define and study functions and measures of finite energy, the non-Archimedean Monge–Ampère operator and investigate the topology of the space of valuations of linear growth, and the behavior of plurisubharmonic functions.

*Boyer and Tonnesen-Friedman* explain how iterated  $S^3$  Sasaki joins can be fit into a categorical framework closely related to the category of Bott orbifold towers. In this framework, they outline various results concerning the existence and non-existence of Sasaki–Einstein, and more generally, constant scalar curvature Sasaki metrics.

Generalizing a landmark result of Berndtsson, *Cao, Guenancia and Păun* establish several formulas for the curvature of the  $L^2$  metric on the direct image of the relative canonical bundle twisted by a holomorphic line bundle endowed with a positively curved metric with analytic singularities. A very explicit lower bound is moreover obtained when the twist is relatively big.

*Collins and Tosatti* define and explore the notion of numerical restricted volume of a pseudoeffective class in a compact Kähler manifold, along a given irreducible subvariety. They relate that notion to the well-known notion of restricted volume of a line bundle on a projective manifold, and conjecture that the non-Kähler locus of a class coincides with its null locus, which they show to hold for classes admitting a Zariski decomposition.

Bergman kernels on smooth compact complex polarized manifolds have been thoroughly studied in the last decades. The growing interest in mildly singular varieties naturally motivates to investigate the asymptotic properties of Bergman currents on compact Kähler spaces. *Coman and Marinescu* study these questions on non-normal algebraic curves.

In a recent breakthrough, Chen and Cheng have shown that coercivity of the Mabuchi K-energy ensures the existence of a constant scalar curvature Kähler metric. *Deruelle and Di Nezza* explain the main steps of the proof of Chen–Cheng, and provide, using a generalized Monge–Ampère capacity, a new and streamlined proof of the crucial uniform  $L^\infty$ -estimate.

The article by *Dineen* studies the connection between  $m$ -subharmonic and  $k$ -plurisubharmonic functions, answering by the negative two questions raised by Sadullaev. In particular the comparison principle does not hold for the twisted Hessian operator, unless the latter equals either the complex Monge–Ampère operator or the Laplacian.

By a famous result of Demailly–Peternell–Schneider the Albanese map of a compact Kähler manifold with nef tangent bundle is a locally trivial fibration, and up to taking a finite étale cover, the fibre is a Fano manifold. *Matsumura* surveys some recent developments of the theory where one aims

at replacing the nefness of the tangent bundle by other conditions (pseudoeffectivity, semi-positive holomorphic sectional curvature, nefness of the relative anticanonical bundle etc).

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