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YOSHINORI HASHIMOTO

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Corrigendum to the paper "Mapping properties of the Hilbert and Fubini–Study maps in Kähler geometry" (*)

Yoshinori Hashimoto (1)

Abstract. — This short note is a corrigendum to the paper in the title.

Résumé. — Cette courte note est un corrigendum à l'article dans le titre.

One of the main results of the paper [2], the surjectivity of the Hilbert map [2, Theorem 1.1], is wrong. A counterexample was provided by Jingzhou Sun [5, Section 3, Example]. Lemma 3.1 of [2], whose proof rests on the surjectivity of the Hilbert map, is also wrong. A counterexample was provided by László Lempert in an email correspondence with the author (see [1, Proposition 4.16]). A precise statement on the closure of the image of the Hilbert map was given by Sun [5, Theorems 1.2–1.3].

The other result in [2, Theorem 1.1], the injectivity of the Fubini–Study map, does hold, although the proof in this paper does not hold since [2, Lemma 3.1] is wrong as stated above. An independent proof of this theorem, together with a more precise statement for the image of the Fubini–Study map, was given by Lempert [4, Theorem 1.1].

The author is so far unable to exactly point out which part of the proof in this paper breaks down, but both counterexamples mentioned above involve basis elements that are redundant in terms of the global generation of the line bundle (see the definition of a rather ample subspace in [4, Section 6]).

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(1) Osaka Metropolitan University, Department of Mathematics, 3-3-138, Sugimoto, Sumiyoshi-ku, Osaka, 558-8585, Japan — yhashimoto@omu.ac.jp

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Article proposé par Jean-François Coulombel.

A corrected, weaker version of [2, Lemma 3.1] is available in an arXiv preprint [3] by the author, but he believes that it is better to separate it as a new work because of its length, and because its statement is different and the proof involves ideas different from the ones used in this paper. Thus its details are not discussed in this corrigendum. The author sincerely apologises for the time it took to prepare this corrigendum, but it was necessary for him to find a corrected version of [2, Lemma 3.1] and prove it.

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