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Global Smooth Solutions of some Quasi-Linear Hyperbolic Systems with Large Data (*)

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RÉSUMÉ. — Nous donnons un critère d'existence de solutions classiques globales en temps pour des systèmes hyperboliques quasi-linéaire quand toutes les valeurs propres sont confondues. Ce critère assure aussi l'existence de solutions globales régulières pour certaines équations de Hamilton-Jacobi. Il permet des données de Cauchy grandes. Il est basé sur l'étude d'une équation de Riccati matricielle.

ABSTRACT. — We give a criterion which ensures the existence of global smooth solution for quasi-linear hyperbolic systems when all the eigenvalues of the system are equal. It provides also the existence of global smooth solution for some Hamilton-Jacobi equations. This criterion allows large Cauchy data. It is based on the study of a matrix valued Riccati equation.

1. Introduction

This work is concerned with the Cauchy problem for quasi-linear hyperbolic systems

$$\frac{\partial}{\partial t}u(t,x) + \sum_{i=1}^{n} A_i(u(t,x)).\frac{\partial}{\partial x_i}u(t,x) = 0, \qquad t > 0,\ x \in I\!R^n,$$

$$u(t=0,x) = u_0(x).$$

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The vector-valued function u maps $[0,T]\times \mathbb{R}^n$ onto $\mathbb{R}^d$ and $A_i(u) \in \mathcal{M}_d(\mathbb{R}^d)$. The matrix $\sum_{i=1}^n \xi_i A_i(u(t,x))$ for $\xi = (\xi_1, ..., \xi_n) \in \mathbb{R}^n$ has real eigenvalues $\lambda_1(u,\xi) \leqslant ... \leqslant \lambda_d(u,\xi)$. Most of the studies deal with strictly hyperbolic systems of conservation laws, i.e. the eigenvalues are assumed to be distinct and the matrix $A_i(u)$ are Jacobian of flux functions $A_i(u) = D_u f_i(u)$, cf [13]. There are only few results about the existence of global in time solution for such systems. The most famous are those of Glimm [5] for $n = 1$ and small Cauchy data and of Diperna [4] for $n = 1, d = 2$ with arbitrarily large data. We refer for a general exposition on the subject and for more detailed references to [13] and [9]. It is also wellknown that in general smoothness of the solutions breakdowns in finite time. However using Ricatti equations, it is possible to find criteria which ensure the existence of global classical solution in the case of an equation $d = 1$ or of a 2 by 2 system of equation in dimension 1 ($n = 1$ and $d = 2$), see [10]. In some sense, this work is an extension of these very classical results. Let us also mention the recent and very interesting result of Grassin, M. and Serre, D. [7] about the existence of smooth solutions for the Euler system of equations.

In this paper we study the very particular case where all the eigenvalues are equal. Then $A_i(u) = c_i(u)Id$ where $c_i(u)$ is a real number and the system becomes

$$\frac{\partial}{\partial t}u(t,x) + \sum_{i=1}^{n} c_i(u(t,x))\frac{\partial}{\partial x_i}u(t,x) = 0, \qquad t > 0,\ x \in \mathbb{R}^n. \tag{1.1}$$

In order to motivate this work, we give two fields where this system arises.

For stellar dynamics physicists ([14]) have proposed pressure-less models. The equations read

$$\frac{\partial}{\partial t}\rho(t,x) + \sum_{i=1}^{3} \frac{\partial}{\partial x_i}(\rho u_i)(t,x) = 0,$$

$$\frac{\partial}{\partial t}\rho u_l(t,x) + \sum_{i=1}^{3} \frac{\partial}{\partial x_i}(\rho u_l u_i)(t,x) = 0, \qquad t > 0,\ x \in \mathbb{R}^3,\ l = 1,2,3.$$

They are obtained from the classical Euler equation of gas dynamics by assuming that the mean velocity $|u|$ of the gas is very large compared to the sound speed. When the velocity is smooth the second equation becomes

$$\frac{\partial}{\partial t}u_l(t,x) + \sum_{i=1}^{3} u_i(t,x)\ \frac{\partial}{\partial x_i}u_l(t,x) = 0, \qquad l = 1,2,3.$$

Let us remark that it is no more a system of conservation laws. We can also consider the relativistic version of these equations which are

$$\frac{\partial}{\partial t}\rho(t,x) + \sum_{i=1}^{3} \frac{\partial}{\partial x_i}(\rho c_i(u))(t,x) = 0,$$

$$\frac{\partial}{\partial t}\rho u_l(t,x) + \sum_{i=1}^{3} \frac{\partial}{\partial x_i}(\rho u_l c_i(u))(t,x) = 0, \qquad l = 1,2,3,$$

or for smooth u $$\frac{\partial}{\partial t}u_l(t,x) + \sum_{i=1}^{3} c_i(u)(t,x)\frac{\partial}{\partial x_i}u_l(t,x) = 0, \qquad l = 1,2,3,$$

where $$c_i(u) = \frac{c_0 u_i}{\sqrt{c_0^2 + |u|^2}}$$ with c_0 the speed of light.

These problems have been investigated in [3, 8, 1]. A rigorous derivation starting from the Euler isentropic equation has been performed in [6] in 1 dimension for classical solutions. Compared with strictly hyperbolic systems a new phenomenon appears. The mass concentration ρ has singularities of the type of Dirac functions just on the shock waves of the velocities u. Therefore the product ρu can not be defined in a classical way. The same phenomenon exists for transport equations with non smooth coefficients [12, 2, 8, 3].

The hyperbolic system (1.1) also appears when you differentiate the Hamilton-Jacobi equation

$$\frac{\partial}{\partial t}\theta(t,x) + f(\nabla_x\theta(t,x)) = 0, \qquad t > 0,\ x \in I\!R^n,$$

where θ is a real-valued function. It is easy to check that if θ is smooth then $u := \nabla_x\theta$ satisfies

$$\frac{\partial}{\partial t}u_l + c(u).\nabla_x u_l = 0, \qquad l = 1, ..., n,$$

with $c(u) = \nabla_u f(u)$ (cf [11, 14, 3]). We point out that our criterion of existence does not impose a convexity assumption on f. Of course there is always a unique viscosity solution [11]. But it is interesting to have a criterion which says when this solution is classical.

In this work we prove that the system (1.1) admits smooth solutions which are global in time for large Cauchy data which satisfy a particular criterion. It is in fact a generalization of the famous property of the Burgers equation in 1 D to be smooth when the initial data is not decreasing. The paper is organized as follows. In the next section we give the basic properties of the system and the main result. The existence Theorem 2.2 is based on a property of the Matrix valued Riccati equation

$$\frac{d}{dt}A + A^2 = 0$$

which is stated in Theorem 2.3. In Section 3 we apply these results for Hamilton-Jacobi equations and we obtain the existence result of Theorem 3.4. Finally Section 4 is devoted to the proof of Theorem 2.3.

2. Basic facts

We study the system

$$\frac{\partial}{\partial t}u(t,x) + \sum_{i=1}^{n} c_i(u(t,x))\frac{\partial}{\partial x_i}u(t,x) = 0, \qquad t > 0,\ x \in \mathbb{R}^n, \tag{2.1}$$

where u is a vector-valued function from $[0,T]\times\mathbb{R}^n$ to $\mathbb{R}^d$ and $c = (c_1, ..., c_n)$ belongs to $C^1(\mathbb{R}^d;\mathbb{R}^n)$.

The system (2.1) is supplemented by a Cauchy data

$$u(t=0,x) = u_0(x). \tag{2.2}$$

In this paper we are only interested in smooth solutions. So we assume

$$u_0 \in [W^{1,\infty}(\mathbb{R}^n)]^d \tag{2.3}$$

We say that T is an existence time if there is a solution of (2.1), (2.2) which belongs to $[W^{1,\infty}((0,T)\times\mathbb{R}^n)]^d$. Let u be such a solution. As usual we define the characteristics $X : [0,T]\times\mathbb{R}^n \to \mathbb{R}^n$ by solving

$$\frac{\partial}{\partial t}X(t,x) = c(u(t,X(t,x)))$$
$$X(0,x) = x$$

These characteristics belong to $[Lip([0,T]\times\mathbb{R}^n)]^n$ and the applications $x \to X(t,x)$ are one to one and onto.

We compute

$$\begin{aligned}\frac{\partial}{\partial t}[u(t,X(t,x))] &= \frac{\partial}{\partial t}u(t,X(t,x)) + \sum_{i=1}^{n}\frac{\partial}{\partial x_i}u(t,X(t,x))\frac{\partial}{\partial t}X_i(t,x)\\ &= \Big(\frac{\partial}{\partial t}u(t,X(t,x)) + \sum_{i=1}^{n}c_i(u)\frac{\partial}{\partial x_i}u\Big)(t,X(t,x)) = 0.\end{aligned}$$

Therefore $u(t, X(t,x)) = u(0, X(0,x)) = u_0(x)$ and the characteristics are given by

$$X(t,x) = x + c(u_0(x))t$$

which can be inverted by

$$x = X(t,x) - c(u(t, X(t,x)))t.$$

So we have

$$u(t, x + c(u_0(x))t) = u_0(x) \tag{2.4}$$

$$u(t, X) = u_0(X - c(u(t, X))t). \tag{2.5}$$

Conversely if $u_0 \in (W^{1,\infty}(\mathbb{R}^n))^d$ then the application $x \to x + c(u_0(x))t$ is one to one and onto as soon as for any $X \in \mathbb{R}^n$, the map $F(x) = X - c(u_0(x))t$ as a unique fixed point. But $D_x F = -t\ D_u c(u_0).D_x u_0$. Therefore for $t < \tau(u_0)$ with

$$\tau(u_0) := \frac{1}{\eta(u_0)}, \quad \eta(u_0) := \|D_u c(u_0(.)).D_x u_0(.)\|_{[L^\infty(\mathbb{R}^n)]^{n\times n}} \tag{2.6}$$

F is a contraction and (2.4) defines a smooth solution. Let

$$M(R) := \sup_{u \in \mathbb{R}^d, |u|<R} |D_u c(u)|,$$

we remark that $\eta(u_0) \leqslant M(\|u_0\|_{(L^\infty(\mathbb{R}^n))^d}) \|Du_0\|_{(L^\infty(\mathbb{R}^n))^{d\times n}} < \infty$ and $\tau(u_0) > 0$. We summarize these results in

THEOREM 2.1.— *Let u_0 be an initial data which satisfies (2.3). Then the problem (2.1), (2.2) has a smooth solution on $[0, \tau(u_0))$ where $\tau(u_0) > 0$ is defined by (2.6). Conversely if $T > 0$ is an existence time, then $\forall t \in [0, T]$ the map $x \to x + u_0(x)\ t$ is one to one and onto and the unique smooth solution is given by (2.4).*

COROLLARY 2.1.— *If there exists a function $\alpha \in L^\infty_{loc}([0, \infty))$ such that for every time t of existence of the smooth solution u of (2.1), (2.2) we have $\eta(u(t,.)) \leqslant \alpha(t)$ then this solution is global in time.*

Indeed if the maximal existence time $T^\star$ is finite then for every $T < T^\star$ we have by using the preceding Theorem with the Cauchy data $u(T,.)$ at time $T : T^\star \geqslant T + \tau(u(T,.))$. By taking the limit as $T \to T^\star$ we obtain that $\text{lim-sup}_{T\to T^\star, T<T^\star} \tau(u(T,.)) = 0$, $\text{lim-inf}_{T\to T^\star, T<T^\star} \eta(u(T,.)) = \infty$.

Let K_R be the complex domain defined by

$$\begin{aligned} K_R := \{\lambda \in \mathbb{C} / Re(\lambda) \geqslant 0 \text{ and } |\lambda| \leqslant R \text{ or} \\ Re(\lambda) < 0 \text{ and } (|\lambda + iR/2| \leqslant R/2 \text{ or } |\lambda - iR/2| \leqslant R/2)\} \end{aligned} \tag{2.7}$$

(see Figure 1). We denote by $Sp(M)$ the set of all eigenvalues of the matrix M. The main result of this paper is

THEOREM 2.2. — *Let us assume that for some $R > 0$ the initial data satisfies (2.3) and*

$$\forall x \in \mathbb{R}^n, \quad Sp(\ D_u c(u_0(x)).D_x u_0(x)\ ) \subset K_R$$

then the smooth solution given by Theorem 2.1 is global in time.

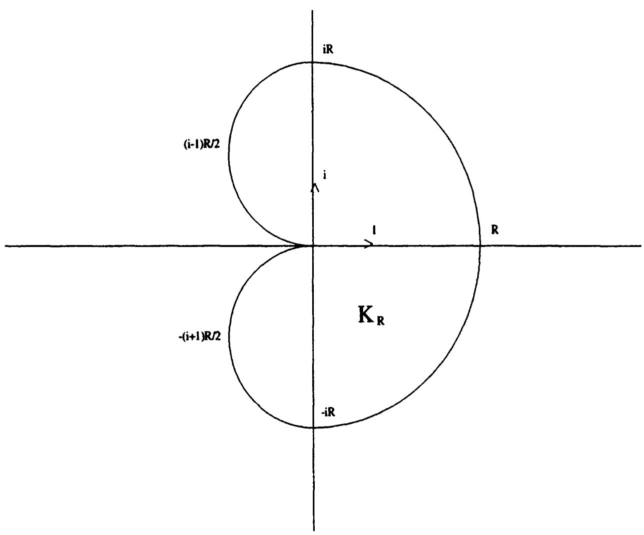

Fig 1 The complex domain K_R

To prove this theorem we need

LEMMA 2.1. — *Let $T^\star$ the maximal existence time of the smooth solution u given in Theorem 2.1. Then the matrix*

$$A(t,x) := D_u c(u(t, x + c(u_0(x))t)).D_x u(t, x + c(u_0(x))t) \tag{2.8}$$

satisfies the Riccati equation

$$\frac{\partial}{\partial t} A(t,x) + A^2(t,x) = 0, \qquad \textit{for } t \in [0, T^\star),\ x \in \mathbb{R}^n. \tag{2.9}$$

Proof. — We differentiate (2.4) w.r.t. x and we get

$$D_x u(t, x + c(u_0(x))t)\ (Id + tA(0,x)) = D_x u_0(x) \tag{2.10}$$

We multiply by $D_u c(u(t, x + c(u_0(x))t)) = D_u c(u_0(x))$ and we obtain

$$A(t,x).(Id + tA(0,x)) = A(0,x).$$

We differentiate w.r.t. time, we get

$$\frac{\partial}{\partial t}A(t,x).(Id + tA(0,x)) + A(t,x).A(0,x) = 0$$

which with the previous relation becomes

$$(\frac{\partial}{\partial t}A(t,x) + A^2(t,x)).(Id + tA(0,x)) = 0.$$

But starting with the Cauchy data $u(s,x)$ at time s we also have for any $t > s \geqslant 0$ with $t < T^\star$

$$(\frac{\partial}{\partial t}A(t,x) + A^2(t,x)).(Id + (t-s)A(s,x)) = 0.$$

For s fixed and $t - s$ sufficiently small the matrix $(Id + (t-s)A(s,x))$ is invertible. So A satisfies (2.9) on any interval of the form $(s, s+\epsilon(s))$ with $\epsilon(s) > 0$ and $s \in [0, T^\star)$ which ends the proof. □

To prove Theorem 2.2 we remark that it is enough to bound $|A(t,x)|$ because of Corollary 2.1 and of the fact that $x \to x + t\ u_0(x)$ is one to one and onto for $t \leqslant T^\star$. But it is an immediate consequence of Lemma 2.1 and

THEOREM 2.3.— *Let $A_0 \in \mathcal{M}_n(\mathbb{C})$ with $Sp(A_0) \subset K_R$ where K_R is defined by (2.7). Then the matrix valued Riccati equation*

$$\frac{dA}{dt}(t) + A(t)^2 = 0, \qquad A(0) = A_0 \tag{2.11}$$

has a solution on $[0, \infty)$. Moreover for every $R > 0$, $M > 0$, there exists a continuous function $\alpha_{M,R} : [0,\infty) \to [0,\infty)$ for which

$$\forall A_0 \in K_R, \text{ with } |A_0| \leqslant M \qquad |A(t)| \leqslant \alpha_{M,R}(t). \tag{2.12}$$

The proof of this Theorem is postponed to Section 4.

3. Hamilton-Jacobi equations

In this section we study the Hamilton-Jacobi equation

$$\frac{\partial}{\partial t}\theta(t,x) + f(\nabla_x\theta(t,x)) = 0, \qquad t > 0,\ x \in \mathbb{R}^n, \tag{3.1}$$

$$\theta(0,x) = \theta_0(x), \qquad x \in \mathbb{R}^n, \tag{3.2}$$

where θ, θ_0 are real functions. We assume that $f \in C^2(\mathbb{R}^n; \mathbb{R})$ and $\theta_0 \in W^{2,\infty}(\mathbb{R}^n)$. We say that θ is a smooth solution of (3.1) if $\theta \in W^{2,\infty}([0,T] \times \mathbb{R}^n)$ for any time $T > 0$ and (3.1) holds everywhere. By differentiating (3.1) w.r.t. x and using $\frac{\partial^2}{\partial x_i \partial x_j}\theta = \frac{\partial^2}{\partial x_j \partial x_i}\theta$ we obtain that $u := \nabla_x \theta$ satisfies (2.1) with $c_i(u) := \partial_i f(u)$. So if $D^2 f(\nabla_x \theta_0) D_x^2 \theta_0$ belongs to K_R we can apply Theorem 2.2. But of course $D^2 f(\nabla_x \theta_0) D_x^2 \theta_0$ is a real symmetric matrix so the eigenvalues are real. We obtain

THEOREM 3.4.— *Assume that* $\theta_0 \in W^{2,\infty}(\mathbb{R}^n)$ *and* $D^2 f(\nabla_x \theta_0(x)) D_x^2 \theta_0(x)$ *is a non negative matrix for every* $x \in \mathbb{R}^n$*, then the problem (3.1), (3.2) has a unique smooth solution which is global in time.*

Proof.— By mean of Theorem 2.2 we solve (2.1), (2.2) for the initial data $u_0 := \nabla_x \theta_0$ with $c := \nabla_u f$. Now we define

$$\theta(t,x) := \theta_0(x) - \int_0^t f(u(s,x))ds.$$

We have

$$\frac{\partial}{\partial t}\theta(t,x) + f(u(t,x)) = 0$$

so it only remains to prove that $u = \nabla_x \theta$. By differentiating the last equation with respect to x_i we have

$$\frac{\partial}{\partial t}\frac{\partial}{\partial x_i}\theta(t,x) + \partial_j f(u(t,x))\frac{\partial}{\partial x_i}u_j(t,x) = 0 \tag{3.3}$$

Now we show that $D_x u$ is symmetric. We use the notations of Section 2. For any time $t \geqslant 0$ the matrix $Id + tA(0,x)$ is positive definite since $A(0,x)$ is non negative. Then (2.10) gives

$$D_x u(t, x + c(u_0(x))t) = (Id + tA(0,x))^{-1} D_x u_0(x) = (Id + tA(0,x))^{-1} D_x^2 \theta_0(x).$$

Therefore the matrix $D_x u(t,x)$ is symmetric.

Then $\frac{\partial}{\partial x_i}u_j(t,x) = \frac{\partial}{\partial x_j}u_i(t,x)$ and (3.3) yields

$$\frac{\partial}{\partial t}\frac{\partial}{\partial x_i}\theta(t,x) + \partial_j f(u(t,x))\frac{\partial}{\partial x_j}u_i(t,x) = \frac{\partial}{\partial t}\Big(\frac{\partial}{\partial x_i}\theta(t,x) - u_i(t,x)\Big) = 0.$$

Since we have equality at $t = 0$ it ends the proof. $\square$

4. The matrix valued Riccati equation

This Section is devoted to the study of the equation (2.11) and to the proof of Theorem 2.3. We denote by $\lambda_1^0 \leqslant \lambda_2^0 \leqslant \ldots \leqslant \lambda_n^0$ the eigenvalues of A_0. Since $\lambda_i^0 \in K_R$ the eigenvalues $1 + t\lambda_i^0$ of the matrix $Id + tA_0$ do not vanish. So it is invertible. Let us define $A(t) := A_0\big(Id + tA_0\big)^{-1}$, we immediately check that $A(t)$ solves (2.11). Then the solution of this O.D.E. is global in time.

The eigenvalues of $A(t)$, $\lambda_i(t)$, $i = 1, \ldots n$ are given by

$$\lambda_i(t) = \frac{\lambda_i^0}{1 + t\lambda_i^0}. \tag{4.1}$$

The trajectories defined by (4.1) are segment of circles centered on the imaginary axis and passing by 0. The eigenvalues move clockwise along circles above the real axis and counter clockwise below (see Figure 2). It follows that K_R defined by (2.7) is an invariant domain for positive time, w.r.t. the spectrum of A. We have

$$Sp(A(t)) \subset K_R, \qquad \forall t \geqslant 0.$$

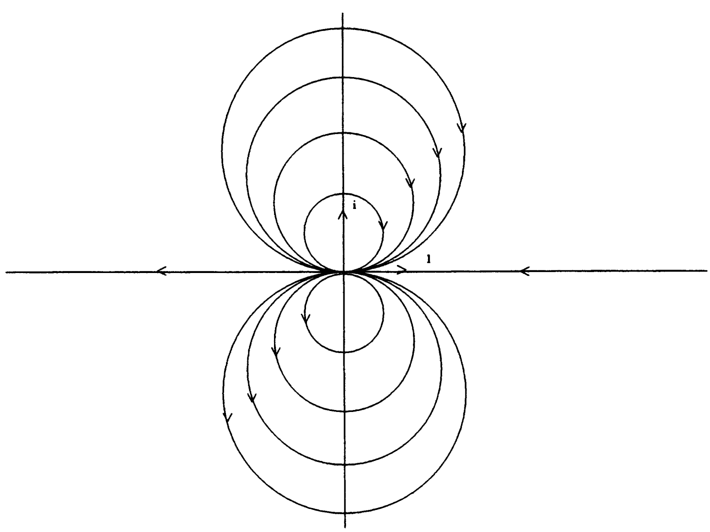

Fig 2 Trajectories of eigenvalues

In order to complete the proof of Theorem 2.3, it remains to establish the existence of the function $\alpha_{M,R}(t)$. We begin with

LEMMA 4.1. — *Let $\mathcal{K}$ be a closed subset of a Banach X and $(S(t))_{t\geqslant 0}$ a semi-group defined on $\mathcal{K}$ which is pointwise bounded, i.e.*

$$\forall x_0 \in \mathcal{K},\ \forall T \geqslant 0, \qquad \sup_{0\leqslant t\leqslant T}|S(t)x| < \infty,$$

and locally continuous, i.e.

$$\forall R > 0, \quad \textit{there exists } \epsilon > 0,\ \forall t \in [0,\epsilon] \qquad S(t) \in \mathcal{C}^0(\mathcal{K}\cap B_R;\mathcal{K})$$

where B_R is the ball of radius R, then $S(t)$ is continuous for every $t \geqslant 0$.

Proof. — For fixed $t_0 \geqslant 0$, $x \in \mathcal{K}$ let $R := 2\sup_{0\leqslant t\leqslant t_0}|S(t)x|$. Let x_n be a sequence in $\mathcal{K}$ which converges towards x. There exists N_1 such that $x_n \in B_R$ for $n \geqslant N_1$. Then for $t \in [0,\epsilon]$ $S(t)x_n \to S(t)x$. In particular there exists N_2 such that $|S(\epsilon)x_n| \leqslant R$ for $n \geqslant N_2$ since $S(\epsilon)x \in B_{R/2}$. By induction we derive the existence of N_k such that $k\epsilon \leqslant t_0 < (k+1)\epsilon$, $|S(k\epsilon)x_n| \leqslant R$ for $n \geqslant N_k$ and $S(k\epsilon)x_n \to S(k\epsilon)x$. Therefore $S(k\epsilon+t)x_n = S(t)S(k\epsilon)x_n \to S(t)S(k\epsilon)x = S(k\epsilon+t)x$ for $t \in [0,\epsilon]$. It remains to choose t such that $k\epsilon + t = t_0$. □

We use this lemma with $X = \mathcal{M}_n(\mathbb{C})$, $\mathcal{K} = \{A \in \mathcal{M}_n(\mathbb{C});\ Sp(A) \subset K_R\}$ and $S(t)A_0 = A(t)$ where A solves (2.11). The local continuity is obtained by using that the norm of $A(t)$ is locally controlled by the solution of the Riccati equation $\frac{dy}{dt}(t) + y(t)^2 = 0,\ y(0) = M$. The pointwise boundedness is a consequence of the continuity w.r.t. time of $t \to A(t)$. We deduce that $S(t)$ is continuous. Then the supremum

$$\alpha_{M,R}(t) := \sup_{A_0\in K_R,\ |A_0|\leqslant M}|S(t)A_0|$$

is a maximum so is bounded. Moreover since the functions $t \to |S(t)A_0|$ are continuous, $\alpha_{M,R}(t)$ is also continuous.

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