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Erratum to "On the geodesic connectedness of simply connected Lorentz surfaces"

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In the statements of Proposition 2.2, Theorem 3.2 (the main result) and Theorem 3.3 in our paper [3], the word "globally hyperbolic" should be replaced by "strongly globally hyperbolic", where a Lorentz surface (M, g) is said to be *strongly globally hyperbolic* if both (M, g) and $(M, -g)$ are globally hyperbolic and any two distinct points of M are causally related in either (M, g) or $(M, -g)$.

Without this strong assumption, one can provide counterexamples to Theorem 3.2. Consider for instance the universal covering of the two-dimensional anti-de Sitter space. This covering may be represented by the strip

$$\widetilde{H_1^2} = \{(x, y) \,/\, -\pi/2 < x < \pi/2\}$$

in $\mathbb{R}^2$ endowed with the Lorentz metric $g = \sec^2 x \left(dx^2 - dy^2\right)$.

It is well known that this space is not geodesically connected (cf. [1], pp. 199-200), and by drawing the two transversal families of null geodesics (called null foliations) one can easily check that $\left(\widetilde{H_1^2}, g\right)$ is not globally hyperbolic. However, the same diagram shows that the Lorentz surface $\left(\widetilde{H_1^2}, -g\right)$ is globally hyperbolic. This gives a simply connected Lorentz surface which is globally hyperbolic but geodesically disconnected.

In the compact case, inspection of the example given in [2], pp. 120-121 shows that the universal covering of the torus $\mathbf{T}^2$ endowed with the Lorentzian metric $g = 2\sin x dx dy + \cos^2 x \left(dx^2 - dy^2\right)$ is globally hyperbolic

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but fails to be geodesically connected. Hence, global hyperbolicity does not imply geodesic connectedness of simply connected Lorentz surfaces even in the compact case.

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